Int. J. Solids Structures Vol. 26, No. 9/10, pp. 1133–1141, 1990
Printed in Great Britain.

0020-7683/90 \$3.00+ .00

RESPONSE OF PRESTRESSED SIMPLY SUPPORTED RECTANGULAR PLATES TO RANDOM TRANSVERSE UNIFORM PRESSURE

P. SEIDE and M. TEHRANIZADEH
Department of Civil Engineering, University of Southern California, Los Angeles, CA 90089-1114, U.S.A.

Abstract—The method of equivalent linearization and the Markov-vector approach are applied to the problem of the single mode nonlinear response of a stretched or compressed simply supported plate subjected to random uniform pressure. Closed form results are obtained for the latter solution in terms of parabolic cylinder functions which are tabulated and whose asymptotic forms are known. RMS values of maximum deflection and stress calculated by the two methods are compared and are found to be in excellent agreement provided the plate has not buckled. For buckled plates the two methods yield values which differ significantly for low values of loading spectral density.

EQUATIONS OF EQUILIBRIUM AND COMPATIBILITY

The equation of motion of a flat plate undergoing moderately large deformations is given by the Von Karman–Mindlin equation

$$\rho h\frac{\partial^2 w}{\partial t^2}+2c\rho h\frac{\partial w}{\partial t}+D\nabla^4 w-\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2}-2\frac{\partial^2 F}{\partial x\,\partial y}\frac{\partial^2 w}{\partial x\,\partial y}+\frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right)=p \tag{1}$$

where F is an Airy stress function such that

$$N_x=\frac{\partial^2 F}{\partial y^2} \tag{2a}$$

$$N_y=\frac{\partial^2 F}{\partial x^2} \tag{2b}$$

$$N_{xy}=\frac{\partial^2 F}{\partial x\,\partial y} \tag{2c}$$

and inplane and rotary inertia have been neglected.

Compatibility of inplane displacements is assured by satisfaction of the equation

$$\nabla^4 F+Eh\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left(\frac{\partial^2 w}{\partial x\,\partial y}\right)^2\right]=0. \tag{3}$$

The rectangular plate (Fig. 1) is assumed to be simply supported on all four edges so that

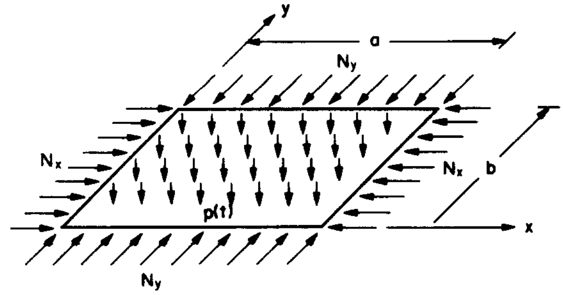

Fig. 1. Plate subjected to inplane loads and random uniform transverse pressure.

$$w = M_x = 0 \quad x = 0, a \tag{4a}$$

$$w = M_y = 0 \quad y = 0, b \tag{4b}$$

with the bending and twisting moments given by

$$M_x = D\left(\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right) \tag{5a}$$

$$M_y = D\left(\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right) \tag{5b}$$

$$M_{xy} = D(1-\nu)\frac{\partial^2 w}{\partial x \, \partial y}. \tag{5c}$$

In addition the edges are assumed to have undergone uniform inplane displacements prior to the imposition of transverse loading and to remain fixed in position thereafter. Inplace edge shear stresses are assumed to vanish, however. Thus the following edge conditions also apply:

$$N_{xy} = 0 \quad x = 0, a; \; y = 0, b \tag{6}$$

$$u = 0 \quad x = 0, \quad u = -u_0 \quad x = a \tag{7a}$$

$$v = 0 \quad y = 0, \quad v = -v_0 \quad y = b \tag{7b}$$

with the inplane displacements given by

$$\frac{\partial u}{\partial x} = \frac{N_x - \nu N_y}{Eh} - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{8a}$$

$$\frac{\partial v}{\partial y} = \frac{N_y - \nu N_x}{Eh} - \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \tag{8b}$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 2(1+\nu)\frac{N_{xy}}{Eh} - \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}. \tag{8c}$$

SOLUTION OF THE EQUATIONS

The deflection function is assumed to be given by

$$w = h \sum_{m=1,3,5,\ldots}^{\infty} \sum_{n=1,3,5,\ldots}^{\infty} \bar{w}_{mn}(t) \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}. \tag{9}$$

Then the stress function may be obtained from eqn (3) as

$$F = -\tfrac{1}{2}(N_{x0}y^2 + N_{y0}x^2) + \frac{\pi^2}{16(1-\nu^2)}\frac{Eh^3}{b^2} \sum_{m=1,3,5,\ldots}^{\infty} \sum_{n=1,3,5,\ldots}^{\infty} \bar{w}_{mn}^2 \cdot \left[\left(\nu\frac{m^2}{\beta^2} + n^2\right)x^2 + \left(\frac{m^2}{\beta^2} + \nu n^2\right)y^2\right] - \frac{Eh^3\beta^2}{8} \sum_{r=0,2,4,\ldots}^{\infty} \sum_{s=0,2,4,\ldots}^{\infty} \bar{\Phi}_{rs} \cos\frac{r\pi x}{a}\cos\frac{s\pi y}{b}. \tag{10}$$

The coefficients $\bar{\Phi}_{rs}$ are quadratic functions of $\bar{w}_{mn}$ and are identical with those of Seide (1978), and

$$N_{x0} = \frac{Eh}{1-\nu^2}\left(\frac{u_0}{a} + \nu\frac{v_0}{b}\right) \tag{11a}$$

$$N_{y0} = \frac{Eh}{1-\nu^2}\left(\frac{v_0}{b} + \nu\frac{u_0}{a}\right) \tag{11b}$$

$$\beta = a/b. \tag{11c}$$

The stress function satisfies the edge conditions on shearing stress and on required edge displacements.

When the assumed deflection function and the associated stress function are substituted into eqn (1) and the Galerkin method is used, the time variation of the displacement coefficients are found to be governed by the following differential equations

$$\frac{d^2\bar{w}_{ij}}{d\bar{t}^2} + 2\bar{c}\frac{d\bar{w}_{ij}}{d\bar{t}} + \bar{\omega}_{ij}^2\bar{w}_{ij} + \bar{g}_{ij}(\bar{w}_{mn}) = \frac{1}{ij}\bar{p}(t) \tag{12}$$

$$m = 1, 3, 5, \ldots, \quad n = 1, 3, 5, \ldots$$

with $\bar{g}_{ij}$ a cubic function of the deflection coefficients $\bar{w}_{mn}$ (Seide, 1978). For a one term approximation

$$\bar{g}_{11} = 3\left[\frac{3-\nu^2}{4}\left(\frac{1}{\beta^4}+1\right) + \frac{\nu}{\beta^2}\right]\bar{w}_{11}^3. \tag{13}$$

The nondimensional quantities in the differential equations are defined by

$$\bar{t} = \left(\frac{\pi^4 D}{\rho h b^4}\right)^{1/2} t \tag{14a}$$

$$\bar{c} = \left(\frac{\rho h b^4}{\pi^4 D}\right)^{1/2} c \tag{14b}$$

$$k_{x0} = \frac{N_{x0} b^2}{\pi^2 D} \tag{14c}$$

$$k_{y0} = \frac{N_{y0} b^2}{\pi^2 D} \tag{14d}$$

$$\bar{p} = \frac{16 p b^4}{\pi^6 D h}. \tag{14e}$$

The small deflection frequency parameter is given by

$$\bar{\omega}_{ij}^2 = \left(\frac{i^2}{\beta^2} + j^2\right)^2 - \left(k_{x0}\frac{i^2}{\beta^2} + k_{y0} j^2\right). \tag{15}$$

Equation (12) comprises a set of coupled nonlinear equations which can be solved only approximately. In the present problem the loading p is a random function of time so that the time averages rather than detailed time distributions are required.

EQUIVALENT LINEARIZATION

One of the approximate methods of solution is that of equivalent linearization wherein the coupled nonlinear equations of motion are replaced by the uncoupled set of equivalent linearized equations.

$$\frac{d^2\bar{w}_{ij}}{d\bar{t}^2} + 2\bar{c}\frac{d\bar{w}_{ij}}{d\bar{t}} + \bar{\omega}_{ij_{EQ}}^2 \bar{w}_{ij} = \frac{1}{ij}\bar{p}(\bar{t}). \tag{16}$$

$$i = 1, 3, 5, \ldots$$
$$j = 1, 3, 5, \ldots$$

The mean-square error of the linearized equations is defined by

$$\langle e_{ij}^2 \rangle = \langle [(\bar{\omega}_{ij}^2 - \bar{\omega}_{ij_{EQ}}^2)\bar{w}_{ij} + \bar{g}_{ij}(\bar{w}_{mn})]^2 \rangle. \tag{17}$$

$$i = 1, 3, 5, \ldots$$
$$j = 1, 3, 5, \ldots$$

The equivalent frequency is chosen so as to minimize the mean-square error yielding

$$\bar{\omega}_{ij_{EQ}}^2 = \bar{\omega}_{ij}^2 + \frac{\langle \bar{w}_{ij}\bar{g}_{ij}(\bar{w}_{mn}) \rangle}{\langle \bar{w}_{ij}^2 \rangle}$$
$$= \bar{\omega}_{ij}^2 + \sum_{m,n,p,q,r,s} b_{ijmnpqrs} \frac{\langle \bar{w}_{ij}\bar{w}_{mn}\bar{w}_{pq}\bar{w}_{rs} \rangle}{\langle \bar{w}_{ij}^2 \rangle}. \tag{18}$$

A procedure similar to that of Seide (1976) gives for uniform pressure having a "white noise" (constant spectral density) time variation

$$\bar{\omega}_{ij_{EQ}}^2 = \bar{\omega}_{ij}^2 + \sum_{m,n,p,q,r,s} b_{ijmnpqrs} \frac{ij}{mnpqrs} \cdot \left(\frac{\bar{S}_{ijmn}\bar{S}_{pqrs} + \bar{S}_{ijpq}\bar{S}_{mnrs} + \bar{S}_{ijrs}\bar{S}_{mnpq}}{\bar{S}_{ijij}} \right) \tag{19}$$

with

$$\bar{S}_{tuvw} = \bar{S}_{vwtu}$$
$$= \frac{8\pi \bar{S}_0 \bar{c}}{(\bar{\omega}_{tu_{EQ}}^2 - \bar{\omega}_{vw_{EQ}}^2)^2 8\bar{c}^2 (\bar{\omega}_{tu_{EQ}}^2 + \bar{\omega}_{vw_{EQ}}^2)} \tag{20}$$

and

$$\bar{S}_0 = \frac{256 b^6 S_0}{\pi^{10}(D\rho h)^{1/2} D h^2}. \tag{21}$$

Here S_0 is the spectral density of the loading p.

The time or ensemble mean-square displacement is now given by

$$\langle \bar{w}_{\max}^2 \rangle = \tilde{E}\left[\left(\frac{w}{h} \right)_{\substack{x=a/2\\ y=b/2}} \right]$$
$$= \sum_{i=1,3,5,\ldots}^{\infty} \sum_{j=1,3,5,\ldots}^{\infty} \sum_{m=1,3,5,\ldots}^{\infty} \sum_{n=1,3,5,\ldots}^{\infty} \frac{(-1)^{(i+j+m+n)/2}}{ijmn} \bar{S}_{ijmn}. \tag{22}$$

Although the differential equations have been linearized, the procedure involves the solution of the set of nonlinear algebraic equations given by eqns (19) and (20). Calculations (Tehranizadeh, 1987) indicate that a single term in the infinite series is sufficient.

Thus, for the unbuckled and buckled plates respectively

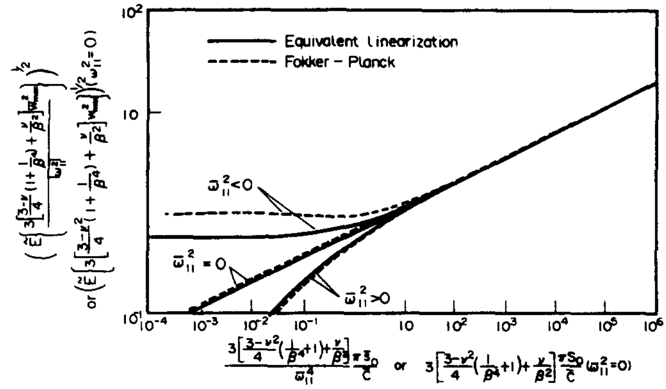

Fig. 2. Comparison of calculations of RMS center deflections.

$$\bar{E}\left(\frac{3\left[\frac{3-\nu^2}{4}\left(1+\frac{1}{\beta^4}\right)+\frac{\nu}{\beta^2}\right]\bar{w}_{\max}^2}{|\bar{\omega}_{11}^2|}\right)=\frac{\bar{S}/\bar{\omega}_{11}^4}{\sqrt{1+6\bar{S}/\omega_2^4}+1}(\bar{\omega}_{11}^2>0) \tag{23a}$$

$$=\frac{\bar{S}/\bar{\omega}_{11}^4}{\sqrt{1+6\bar{S}/\bar{\omega}_{11}^4}-1}(\bar{\omega}_{11}^2<0) \tag{23b}$$

with

$$\bar{S}=3\left[\frac{3-\nu^2}{4}\left(1+\frac{1}{\beta^4}\right)+\frac{\nu}{\beta^2}\right]\pi\bar{S}_0/\bar{c}. \tag{24}$$

If the axial load is at the critical value so that $\bar{\omega}_{11}^2$ is equal to zero

$$\bar{E}\left(3\left[\frac{3-\nu^2}{4}\left(1+\frac{1}{\beta^4}\right)+\frac{\nu}{\beta^2}\right]\bar{w}_{\max}^2\right)=\sqrt{\frac{\bar{S}}{6}}. \tag{25}$$

The RMS values of center deflection given by eqns (23)–(25) are shown by the solid curves in Fig. 2.

FOKKER–PLANCK EQUATION

With the use of a single mode ($\bar{w}_{11}\neq 0$, $w_{ij}=0$ otherwise), the equation of motion becomes

$$\frac{\mathrm{d}^2\bar{w}_{11}}{\mathrm{d}\bar{t}^2}+2\bar{c}\frac{\mathrm{d}\bar{w}_{11}}{\mathrm{d}\bar{t}}+\bar{\omega}_{11}^2\bar{w}_{11}+3\left[\frac{3-\nu^2}{4}\left(\frac{1}{\beta^4}+1\right)+\frac{\nu}{\beta^2}\right]\bar{w}_{11}^3=\bar{p}(\bar{t}). \tag{26}$$

If $\bar{p}(\bar{t})$ is Gaussian white noise, then the transition probability function q is governed by the Fokker–Planck equation (Lin, 1967)

$$\frac{\partial q}{\partial\bar{t}}+\frac{\partial}{\partial\bar{w}_{11}}(\dot{\bar{w}}_{11}q)-\frac{\partial}{\partial\dot{\bar{w}}_{11}}\left\{2\bar{c}\dot{\bar{w}}_{11}+\bar{\omega}_{11}^2\bar{w}_{11}+3\left[\frac{3-\nu^2}{4}\left(\frac{1}{\beta^4}+1\right)+\frac{\nu}{\beta^2}\right]\bar{w}_{11}^3\right\}q-\pi\bar{S}_0\frac{\partial^2 q}{\dot{\bar{w}}_{11}^2}=0. \tag{27}$$

If, furthermore, the response is stationary the solution of the equation may be obtained as (Lin, 1976)

$$q = q_1(\bar{w}_{11})q_2(\dot{\bar{w}}_{11}) \tag{28a}$$

with

$$q_1(\bar{w}_{11}) = C_1 \exp\left(-\frac{2\bar{c}}{\pi\bar{S}_0}\left\{\frac{1}{2}\bar{\omega}_{11}^2\bar{w}_{11}^2 + \frac{3}{4}\left[\frac{3-\nu^2}{4}\left(1+\frac{1}{\beta^4}\right)+\frac{\nu}{\beta^2}\right]\bar{w}_{11}^4\right\}\right) \tag{28b}$$

$$q_2(\dot{\bar{w}}_{11}) = C_2 \exp\left(-\frac{\bar{c}}{\pi\bar{S}_0}\dot{\bar{w}}_{11}^2\right) \tag{28c}$$

where C_1 and C_2 are normalization factors such that

$$\int_{-\infty}^{\infty} q_1(\bar{w}_{11})\,\mathrm{d}\bar{w}_{11} = \int_{-\infty}^{\infty} q_2(\dot{\bar{w}}_{11})\,\mathrm{d}\bar{w}_{11} = 1. \tag{28d}$$

We note the following integrals (Gradshteyn and Ryzhik, 1980 and Abramowitz and Stegun, 1964).

$$\int_{-\infty}^{\infty} \mathrm{e}^{-(\gamma x^2+\mu x^4)}\,\mathrm{d}x = \frac{1}{(2\mu)^{1/4}}\mathrm{e}^{\gamma^2/8\mu}\begin{cases}\sqrt{\pi}U(0,\gamma/\sqrt{2\mu}) & \gamma \geqslant 0\\ \pi V(1,-\gamma/\sqrt{2\mu}) & \gamma \leqslant 0\end{cases} \tag{29a}$$

$$\int_{-\infty}^{\infty} x^2\mathrm{e}^{-(\gamma x^2+\mu x^4)}\,\mathrm{d}x = \frac{1}{(2\mu)^{3/4}}\mathrm{e}^{\gamma^2/8\mu}\begin{cases}\frac{\sqrt{\pi}}{2}U\left(1,\frac{\gamma}{\sqrt{2\mu}}\right) & \gamma \geqslant 0\\ \pi V\left(1,\frac{-\gamma}{\sqrt{2\mu}}\right) & \gamma \leqslant 0\end{cases} \tag{29b}$$

$$\int_{-\infty}^{\infty} \mathrm{e}^{-\gamma x^2}\,\mathrm{d}x = \sqrt{\pi/\gamma} \tag{29c}$$

and U, V are parabolic cylinder functions. With the aid of eqns (28) and (29) the mean-square central deflection can be found to be given by (Lin, 1967)

$$E(\bar{w}_{11}^2)\int_{-\infty}^{\infty} \bar{w}_{11}^2 q_1(\bar{w}_{11})\,\mathrm{d}\bar{w}_{11} \tag{30a}$$

or

$$\tilde{E}\left\{3\left[\frac{3-\nu^2}{4}\left(\frac{1}{\beta^4}+1\right)+\frac{\nu}{\beta^2}\right]\bar{w}_{\max}^2\right\} = \bar{S}^{1/2}\begin{cases}\frac{1}{2}\frac{U\left(1,\frac{\bar{\omega}_{11}^2}{\sqrt{\bar{S}^{1/2}}}\right)}{U\left(0,\frac{\bar{\omega}_{11}^2}{\sqrt{\bar{S}^{1/2}}}\right)}(\bar{\omega}_{11}^2 \geqslant 0) & \text{(30a)}\\ \frac{V\left(1,\frac{-\bar{\omega}_{11}^2}{\sqrt{\bar{S}^{1/2}}}\right)}{V\left(0,\frac{-\bar{\omega}_{11}^2}{\sqrt{\bar{S}^{1/2}}}\right)}(\bar{\omega}_{11}^2 \leqslant 0). & \text{(30b)}\end{cases}$$

The values given by eqns (30) are shown by the dashed curves of Fig. 2. It may be seen that the results are in close agreement with those of equivalent linearization when the plate is unbuckled ($\bar{\omega}_{11}^2 \geqslant 0$) or when the loading spectral density is large. For sufficiently large

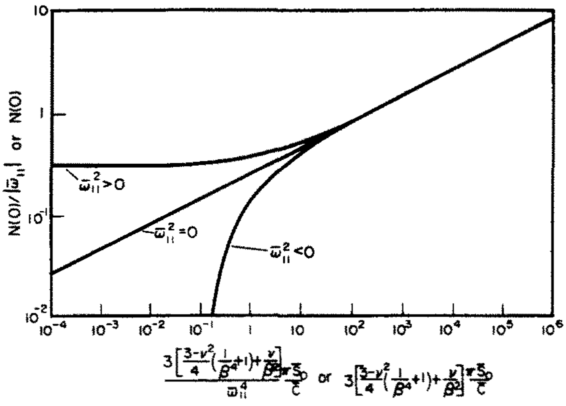

Fig. 3. Variation of zero-crossing frequency with spectral density.

spectral density the RMS deflections are independent of the initial stretching or compression of the plate. The method of equivalent linearization is seen to be seriously in error when the nonlinearity of the equation is severe.

The calculated probability function may also be used to obtain the rate of zero crossing of the plate center deflection. The number of crossings per unit time of the desired level $\dot{\bar{w}}_{11} = 0$ is given by (Lin, 1967)

$$N(0) = N^+(0)+N^-(0) = \left[\int_{-\infty}^{\infty} |\dot{\bar{w}}_{11}| q_2(\dot{\bar{w}}_{11})\, d\dot{\bar{w}}_{11}\right] q_1(0)$$

$$= \frac{\bar{S}^{1/4}}{\pi}\exp\left(-\frac{1}{4}\frac{\bar{\omega}_{11}^2}{\bar{S}}\right)\begin{cases} \dfrac{1}{U(0, \bar{\omega}_{11}^2/\bar{S}^{1/2})} & \bar{\omega}_{11}^2 \geqslant 0 \\ \dfrac{1}{\pi^{1/2} V(0, -\bar{\omega}_{11}^2/\bar{S}^{1/2})} & \bar{\omega}_{11}^2 \leqslant 0 \end{cases} \tag{31}$$

These results are shown in Fig. 3 and also indicate that the results are independent of stretching or compression for large values of spectral density.

MEAN-SQUARE STRESSES

While mean-square stresses are known to require the evaluation of many more terms than a single term, a comparison of one term results given by equivalent linearization and the solution of the Fokker–Planck equation may serve to indicate the accuracy of the method of equivalent linearization. One-term membrane and bending stresses at the plate center are given by

$$\frac{\sigma_{mx} b^2 h}{\pi^2 D} = \frac{3}{2}\left(\frac{2-\nu^2}{\beta^2}+\nu\right)\bar{w}_{11}^2 - k_{x0} \tag{32a}$$

$$\frac{\sigma_{my} b^2 h}{\pi^2 D} = \frac{3}{2}\left(2-\nu^2+\frac{\nu}{\beta^2}\right)\bar{w}_{11}^2 - k_{y0} \tag{32b}$$

$$\frac{\sigma_{bx} b^2 h}{\pi^2 D} = 6\left(\frac{1}{\beta^2}+\nu\right)\bar{w}_{11} \tag{32c}$$

$$\frac{\sigma_{by} b^2 h}{\pi^2 D} = 6\left(1+\frac{\nu}{\beta^2}\right)\bar{w}_{11}. \tag{32d}$$

Then mean square maximum stresses are given

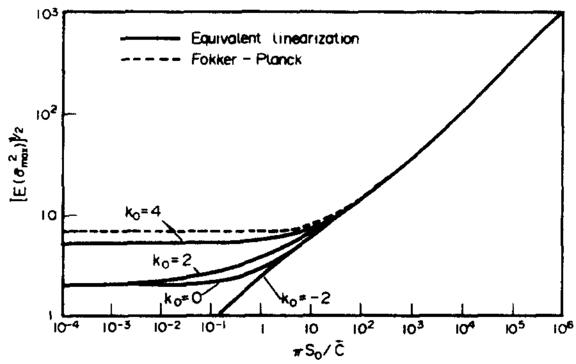

Fig. 4. Comparison of RMS stress in a square plate by two methods of analysis ($k_{x0} = k_{y0} = k_0$; $\nu = 0.3$).

$$\tilde{E}\left[\left(\frac{\sigma_{x_{\max}} b^2 h}{\pi^2 D}\right)^2\right] = k_{x0}^2 + \left[36\left(\frac{1}{\beta^2}+\nu\right)^2 - 3k_{x0}\left(\frac{2-\nu^2}{\beta^2}+\nu\right)\right]\tilde{E}(\bar{w}_{11}^2) + \frac{9}{4}\left(\frac{2-\nu^2}{\beta^2}+\nu\right)^2 \tilde{E}(\bar{w}_{11}^4) \quad (33a)$$

$$\tilde{E}\left[\left(\frac{\sigma_{y_{\max}} b^2 h}{\pi^2 D}\right)^2\right] = k_{y0}^2 + \left[36\left(1+\frac{\nu}{\beta^2}\right)^2 - 3k_{y0}\left(2-\nu^2+\frac{\nu}{\beta^2}\right)\right]E(\bar{w}_{11}^2) + \frac{9}{4}\left(2-\nu^2+\frac{\nu}{\beta^2}\right)^2 E(\bar{w}_{11}^4). \quad (33b)$$

In the case of equivalent linearization $\tilde{E}(\bar{\omega}_{11}^2)$ is given by eqn (24) and

$$E(\bar{w}_{11}^4) = 3E(\bar{w}_{11}^2)^2 \quad (34)$$

while for the solution of the Fokker–Planck equation $E(\bar{w}_{11}^2)$ is given by eqn (30) and

$$E(\bar{w}_{11}^4) = \int_{-\infty}^{\infty} \bar{w}_{11}^4 q_1(\bar{w}_{11})\,\mathrm{d}\bar{w}_{11} = \frac{\pi\bar{S}_0/\bar{c}}{3\left[\frac{3-\nu^2}{4}\left(1+\frac{1}{\beta^4}\right)+\frac{\nu}{\beta^2}\right]}\left[\begin{array}{ll} \dfrac{3}{4}\dfrac{U\left(2,\dfrac{\bar{\omega}_{11}^2}{\bar{S}^{1/2}}\right)}{U\left(0,\dfrac{\bar{\omega}_{11}^2}{\bar{S}^{1/2}}\right)} & \bar{\omega}_{11}^2 \geqslant 0 \\ \dfrac{V\left(2,\dfrac{\bar{\omega}_{11}^2}{\bar{S}^{1/2}}\right)}{V\left(0,\dfrac{\bar{\omega}_{11}^2}{\bar{S}^{1/2}}\right)} & \bar{\omega}_{11}^2 \leqslant 0. \end{array}\right. \quad (35)$$

Results are shown in Fig. 4 for a square plate with $\nu = 0.3$ and

$$k_{x0} = k_{y0} = k_0 \quad (36)$$

and indicate that the one mode stresses obtained by the two methods are in excellent agreement so long as the plate is unbuckled ($k_0 \leqslant 2$). It may be conjectured that equivalent linearization results using additional terms are accurate. For buckled plates, however, results given by equivalent linearization are unconservative. Accurate stress values in this case would be obtained only by actual test or by numerical simulation.

REFERENCES

Abramowitz, M. and Stegun, I. A. (1964). *Handbook of Mathematical Functions*. Nat. Bur. Stds Appl. Math. Ser. 55, 686–720.

Gradshteyn, I. S. and Ryzhik, I. M. (1980). *Table of Integrals, Series, and Products*. p. 337. Academic Press, New York.

Lin, Y. K. (1967). *Probabilistic Theory of Structural Dynamics*. McGraw-Hill Book Co., 253–267, 293–299.

Seide, P. (1976). Nonlinear stresses and deflections of beams subjected to random time dependent uniform pressure. *ASME J. Eng., Ind.* **98,** 1014–1020.

Seide, P. (1978). Large deflections of prestressed simply supported rectangular plates under uniform pressure. *Int. J. Nonlinear Mech.* **13,** 145–156.

Tehranizadeh, M. (1987) *Response of Stretched or Compressed Simply Supported Rectangular Plates to Random Transverse Uniform Pressure*. Ph.D. Dissertation, University of Southern California, Los Angeles.